Upright Sailing Craft Performance and Optimum Speed to Windward

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A convenient geometric solution to the wind triangle relationships for a sailing craft is developed which is directly related to the aerodynamic and hydrodynamic characteristics of the craft. Maximizing the speed made good to windward is examined, and a relationship between the aerodynamics and hydrodynamics of the craft which bounds the windward performance and is a necessary condition for maximum speed to windward is developed. A brief discussion of windward performance tradeoffs is included based on this result.

Nomenclature

$\mathcal{R}$ = aspect ratio
C_D = drag coefficient, $C_D = D/(\frac{1}{2}\rho V^2 S)$
C_{D0} = profile drag coefficient. For the hull since the coefficients are based on centerboard area, $C_{D0} = [2 + (S_w/S_H)]C_{D_w}$
C_{D_w} = drag coefficient based on wetted area
C_L = lift coefficient, $C_L = L/(\frac{1}{2}\rho V^2 S)$
$\bar{C}_L$ = ratio of lift coefficient to lift coefficient at maximum lift/drag ratio
C_R = resultant force coefficient, $C_R = R/(\frac{1}{2}\rho V^2 S)$
D = drag, force parallel to velocity acting on component, lb
K = factor relating induced drag to square of lift coefficient
L = lift, force perpendicular to velocity acting on component, lb
P = performance parameter, see Eq. (8), $P = [(\rho_w S_H \times C_{RH})/(\rho_A S_s C_{R_s})]^{1/2}$
P_0 = optimum value of P corresponding to maximum lift/drag ratios of sail and hull
P_M = value of performance parameter evaluated at maximum lift/drag ratios of sail and hull
R = resultant force, $R = (L^2 + D^2)^{1/2}$, lb
S_s = sail area, ft^2
S_w = wetted surface area of hull not including centerboard or keel, ft^2
S_H = centerboard area, ft^2
v_B = dimensionless boat velocity, boat velocity divided by true wind velocity
v_R = dimensionless relative wind velocity, relative wind velocity divided by true wind velocity
v_x = component of dimensionless boat velocity perpendicular to true wind, $v_x = v_B \sin\gamma_T$
v_y = component of dimensionless boat velocity parallel to true wind, $v_y = v_B \cos\gamma_T$
V_B = boat velocity, fps
V_{MG} = velocity made good to windward, component of boat velocity in direction of true wind ($V_{MG} = V_B \cos\gamma_T$), fps
V_R = relative wind velocity, fps
V_T = true wind velocity, fps
α = angle of attack of sail
γ_R = angle between relative wind velocity and boat velocity
γ_T = angle between true wind velocity and boat velocity
ϵ = angle between resultant force vector of component and perpendicular to velocity acting on component, $\epsilon = \tan^{-1}(D/L)$
ϵ_p = profile drag/lift ratio at maximum lift/drag ratio
λ = leeway angle, angle between centerline of boat and boat velocity (equal to keel angle of attack)
ρ_A = density of air, slugs per ft^3
ρ_w = density of water, slugs per ft^3
$(\ )_H$ = value of parameter for all components immersed in water (for convenience, referred to as hull)
$(\ )_M$ = value of parameter at maximum lift/drag ratio
$(\ )_0$ = value of parameter for maximum speed to windward
$(\ )_s$ = value of parameter for all components immersed in air (for convienience, referred to as sail)

Presented at the AIAA Ancient Interface VII Symposium, California State College, Long Beach, Calif., Jan. 31, 1976 (no preprints); submitted Sept. 1, 1976; revision received Dec. 6, 1976.
Index category: Marine Vessel Design (including loads).
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Introduction

DETERMINING the best performance of a sailing craft to windward involves understanding complex interactions between the aerodynamics and hydrodynamics of the craft. It was thought to be valuable to examine a simplified analytical model rather than proceeding directly to consider the complete problem. Therefore, this paper discusses maximizing the speed of a sailing craft to windward assuming that the heel angle is zero and that the hull forces depend upon the square of the hull velocity. The first assumption is quite typical of the way in which small boats are sailed. The second assumption, which implies that the Froude number or speed-length ratio effects are neglected, restricts the analysis to sailing in light winds or to hulls of very high fineness ratio.

Geometry of the Wind Triangle

In order to examine the manner in which various aerodynamic and hydrodynamic factors influence the performance of a sailing craft it is first desirable to obtain certain geometrical relationships involved in the wind triangle. Wind triangle refers to the relationships among the various quantities: the true wind velocity V_T; the boat speed V_B; the relative wind velocity V_R; the angle between the true wind direction and the direction the boat is moving γ_T; and the angle between the relative wind direction and the direction the boat is moving γ_R. Figure 1 shows the geometry relating these quantities. The leeway angle λ also is shown, emphasizing the fact that, in general, the craft is moving through the water at some inclination to the centerline of the hull.

Instruments mounted on the craft can measure directly the boat speed V_B; the relative wind speed V_R; and the angle between the direction of the relative wind and the centerline of the boat ($\gamma_R - \lambda$). Sailing to windward in a race, the helmsman attempts to maximize the speed made good to windward V_{MG},

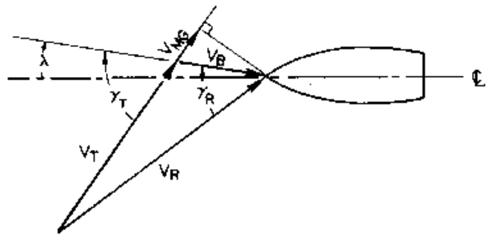

Fig. 1 Wind triangle geometry.

also shown in Fig. 1. Note that the presence of the leeway angle, which is determined by the aerodynamic and hydrodynamic characteristics of the craft, complicates the problem of determining the actual speed made good to windward from the instrumentation usually found on sailing craft because the angle between the direction of the relative wind and the direction the craft is moving γ_R, cannot be measured directly without knowledge of the leeway angle. A wind vane mounted on the craft measures $(\gamma_R - \lambda)$. Although in principle it is possible to measure the leeway angle with similar instrumentation underwater, many practical difficulties are associated with the measurement of this quantity.

The law of cosines can be applied to the geometry of Fig. 1 to obtain relationships between relative and true quantities

$$\frac{\sin\gamma_T}{V_R} = \frac{\sin\gamma_R}{V_T} = \frac{\sin(\gamma_T - \gamma_R)}{V_B} \qquad (1)$$

These relationships can be reorganized so that the wind triangle geometry can be displayed conveniently.

The relative wind velocity V_R and boat velocity V_B are nondimensionalized by the true wind velocity V_T. Introducing the operating condition of the boat with respect to the true wind expressed in terms of the components perpendicular and parallel to the true wind, $v_x = v_B \sin\gamma_T$ and $v_y = v_B \cos\gamma_T$, two equations equivalent to Eq. (1) can be developed

$$\left(v_x + \frac{1}{2}\right)^2 + \left(v_y - \frac{1}{2\tan\gamma_R}\right)^2 = \frac{1}{4} + \frac{1}{4\tan^2\gamma_R} \qquad (2)$$

$$v_x^2 + \left(v_y + \frac{1}{1-(v_R/v_B)^2}\right)^2 = \left(\frac{v_R/v_B}{1-(v_R/v_B)^2}\right)^2 \qquad (3)$$

Equations (2) and (3) can be recognized as equations of circles and the operating condition of the boat is expressed directly in terms of the parameters, the relative wind angle γ_R and the ratio of relative wind velocity to boat velocity V_R/V_B. Equations (2) and (3) are shown graphically in Figs. 2 and 3. The coordinates of these two figures also can be interpreted in polar form, that is, the true wind direction is along the y axis and a radius vector to any point on a particular circle is the ratio of the boat speed to the true wind speed. The angle between the vertical axis and this radius vector is the heading of the craft with respect to the true wind direction.

If the quantities V_R, V_B, and γ_R are known, then these two curves can be readily used to calculate the true wind velocity, and the angle between direction of the true wind and the direction the boat is moving. That is, knowing the relative wind angle γ_R the operating condition of the craft must be on a particular circle of Fig. 2, and knowing the ratio of the relative wind speed to the boat speed V_R/V_B the operating condition of the craft must be on a particular circle of Fig. 3. Overlaying these two curves, the intersection of the two circles is the operating condition of the craft. Again, recall that in a practical sense, difficulty in determination of the speed made good to windward arises because γ_R cannot be measured directly without knowing the leeway angle.

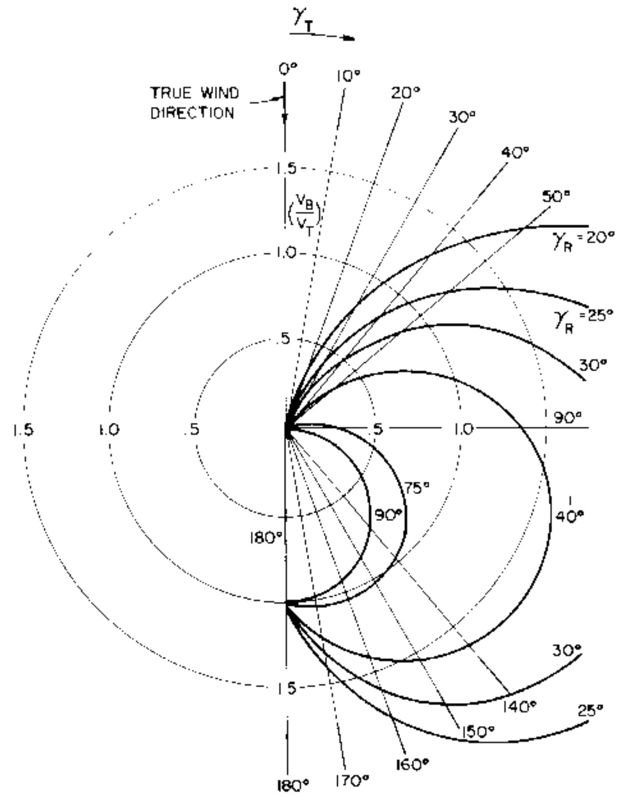

Fig. 2 Wind triangle geometry: lines of constant relative wind angle.

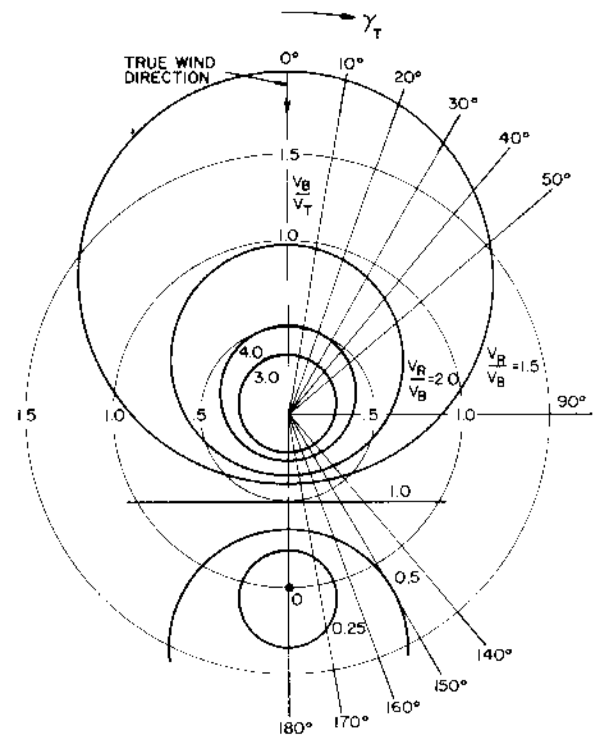

Fig. 3 Wind triangle geometry: lines of constant relative windspeed to boat speed.

Balance of Forces

The performance of a sailing craft now will be examined in relation to the wind triangle geometry discussed. For simplicity, only the case of upright sailing is considered. This analysis therefore is restricted to small boats in which the crew is always acting to maintain the craft at zero heel angle or perhaps to larger boats in light winds. Therefore, only

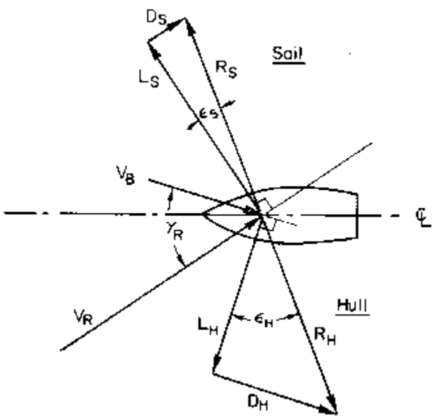

Fig. 4 Equilibrium of sail and hull forces.

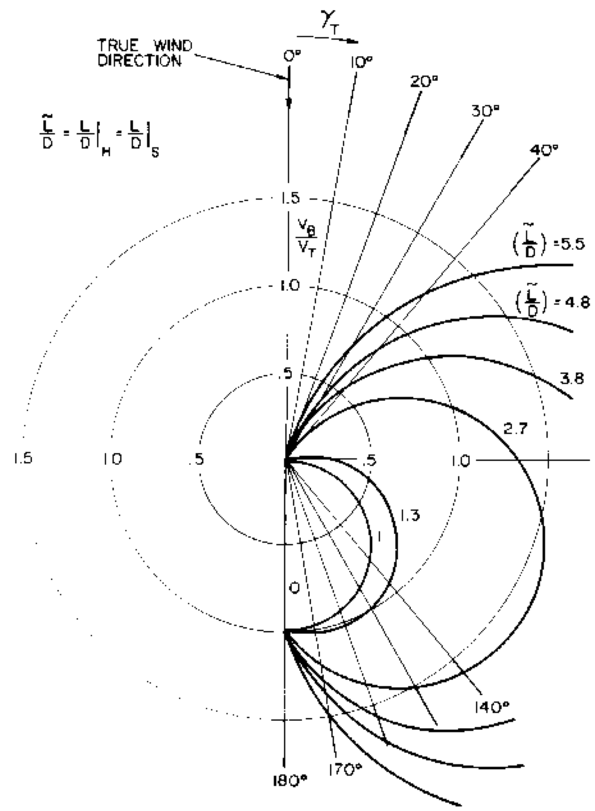

Fig. 5 Force equilibrium: lines of constant lift/drag ratio.

equilibrium in a plane parallel to the surface of the water is considered. The aerodynamic forces acting on the craft due to the action of the relative wind are characterized by a resultant force vector R_s acting at an angle ϵ_s measured downwind from the perpendicular to the direction of the relative wind as shown in Fig. 4. Similarly, the hydrodynamic forces acting on the hull, keel, and rudder are characterized by a resultant force vector R_H acting at an angle ϵ_H measured downstream from the perpendicular to the direction of the boat velocity.

The angles denoted by ϵ_s and ϵ_H are related to the lift/drag ratios of the various components by

$$\epsilon_s = \tan^{-1}(D/L)_s$$

$$\epsilon_H = \tan^{-1}(D/L)_H$$

From Fig. 4, the condition that the forces are balanced in two directions may be expressed conveniently as[1]

$$\gamma_R = \epsilon_H + \epsilon_s \tag{4}$$

$$R_s = R_H \tag{5}$$

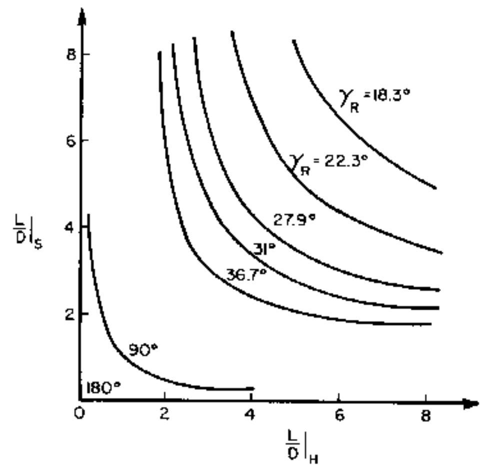

Fig. 6 Relative wind angle as a function of lift/drag ratios of sail and hull.

Relationship (4) indicates that given the lift/drag ratios of the sail, rig, and hull above water ϵ_s and the hull, keel, and rudder ϵ_H, the operating condition of the craft must lie on a particular circle shown in Fig. 2. Figure 5 is identical to Fig. 2 with various values of the lift/drag ratios corresponding to various γ_R shown. The lift/drag ratios indicated in Fig. 5 are for the case in which the aerodynamic components and hydrodynamic components have equal lift/drag ratios. In general,

$$\tan\gamma_R = ((L/D)_s + (L/D)_H)/((LD)_s(L/D)_H - 1)$$

This relationship is shown in Fig. 6.

Figure 5 clearly shows the importance of achieving high lift/drag ratios for good windward performance. Recall that the projection of any point on a circle to the vertical axis gives the speed made good to windward.

Now consider the second equilibrium condition given by Eq. (5). Expressing this relationship in coefficient form

$$\tfrac{1}{2}\rho_A V_R^2 S_s C_{R_s} = \tfrac{1}{2}\rho_w S_H V_B^2 C_{R_H} \tag{6}$$

(for brevity, sail refers to all components of craft acted upon by the air and hull refers to all components of the craft acted on by the water). The hull area used here is the centerboard or keel area since, in sailing to windward, the largest part of the hull force arises from the centerboard or keel as a result of leeway. The drag coefficient of the hull thus depends upon the ratio of wetted area to centerboard area as noted in the Nomenclature. The sail force depends on the air density ρ_A and the relative wind velocity V_R, and the hull force depends upon the water density ρ_w and the boat speed V_B. In general, the resultant force coefficient C_{R_s} depends upon the geometry and angle of attack of the sail, rig, etc. C_{R_H} depends upon the leeway angle and also, in general, the Froude number or speed-length ratio. The Froude number influence will be neglected, thus the wave making resistance of the hull is being neglected and strictly speaking, only sailing in light winds is being examined. A number of conclusions are not influenced by this assumption.

Rearranging Eq. (6)

$$\left(\frac{V_R}{V_B}\right) = \left\{\frac{\rho_w S_H C_{R_H}}{\rho_A S_s C_{R_s}}\right\}^{1/2} \tag{7}$$

If the density ratio ρ_w/ρ_A, the centerboard area S_H, and the sail area S_s are known along with the resultant force coefficient C_{R_H} and C_{R_s}, the operating condition of the craft must be on a specific circle of Fig. 3. Thus, the conditions of force equilibrium can be related simply to the wind triangle

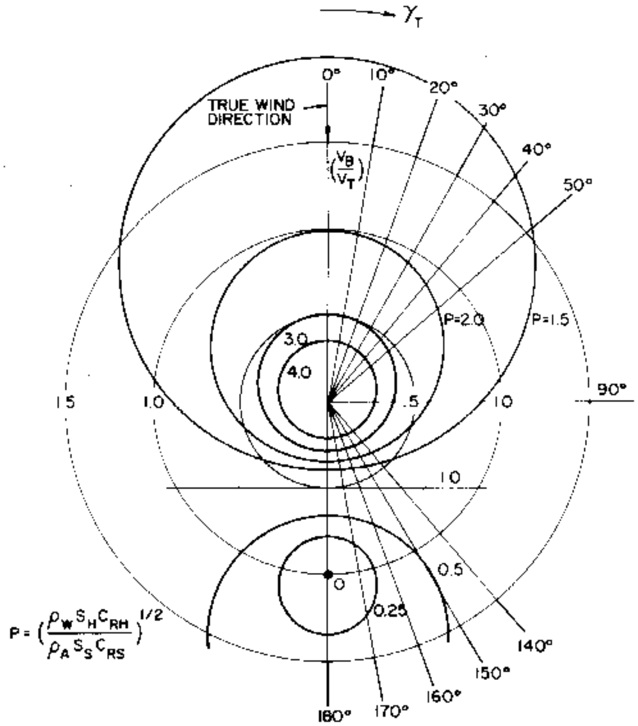

Fig. 7 Force equilibrium: lines of constant P.

geometry. In the following discussion the quantity on the right-hand side of Eq. (7) is denoted by P

$$P \equiv \left\{ \frac{\rho_W S_H C_{R_H}}{\rho_A S_S C_{R_S}} \right\}^{1/2} \tag{8}$$

Figure 7 illustrates the relationship between P and the true quantities, γ_T and V_B/V_T.

Maximum Speed to Windward

The question of obtaining the maximum speed to windward now is examined. The ratio of the speed made good to the true windspeed can be expressed as follows

$$v_{MG} = v_B \cos\gamma_T \tag{9}$$

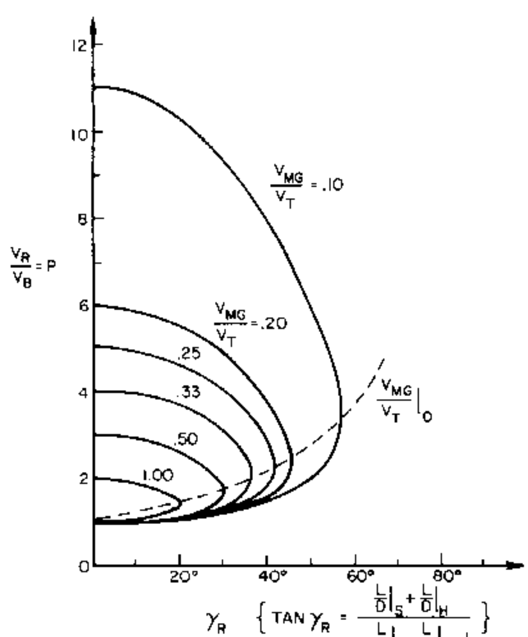

Fig. 8 Speed made good to windward.

Using the wind triangle geometry, the speed made good to windward can be expressed in terms of the ratio of relative windspeed to boat speed V_R/V_B, and the relative wind angle γ_R as

$$\frac{V_{MG}}{V_T} = \frac{(V_R/V_B)\cos\gamma_R - 1}{(V_R/V_B)^2 - 2(V_R/V_B)\cos\gamma_R + 1} \tag{10}$$

Figure 8 shows the relationship between V_R/V_B and γ_R for various constant values of speed made good to windward. Also, from force balance considerations, the speed made good to windward can be related directly to the aerodynamic and hydrodynamic characteristics of the craft from Eqs. (4) and (7).

$$v_{MG} = (P\cos\gamma_R - 1)/(P^2 - 2P\cos\gamma_R + 1) \tag{11}$$

Figure 8 shows that given the maximum lift/drag ratios of the sail and hull $[(\epsilon_S + \epsilon_H)$ is a minimum], there is a particular value of P corresponding to a maximum speed made good to windward. This value of P is given by the point on the graph where the curves of constant v_{MG} have a vertical slope.

The problem of maximizing speed to windward can be stated as follows. Functionally from Eq. (11) the speed made good to windward can be expressed as

$$v_{MG} = v_{MG}(P, \gamma_R) \tag{12}$$

where the quantities P and γ_R depend upon the angle of attack of the sail α and the keel λ or, equivalently, the lift coefficients of each component

$$\gamma_R = \gamma_R(C_{L_S}, C_{L_H}) \tag{13}$$

$$P = P(C_{L_S}, C_{L_H}) \tag{14}$$

Therefore, the speed made good to windward v_{MG} is maximized by the conditions

$$\frac{\partial v_{MG}}{\partial C_{L_S}} = 0 \qquad \frac{\partial v_{MG}}{\partial C_{L_H}} = 0 \tag{15}$$

From Eqs. (12-15)

$$\frac{\partial v_{MG}/\partial P}{\partial v_{MG}/\partial \gamma_R} = -\frac{\partial\gamma_R/\partial C_{L_S}}{\partial P/\partial C_{L_S}} = -\frac{\partial\gamma_R/\partial C_{L_H}}{\partial P/\partial C_{L_H}} \tag{16}$$

Equation (16) gives the relationship between the wind triangle geometry and the hydrodynamics and aerodynamics of the craft which maximizes the speed made good to windward.

From Eq. (10) it may be noted that $\partial v_{MG}/\partial\gamma_R$ is always negative, whereas $\partial v_{MG}/\partial P$ may be positive, negative, or zero. Therefore, at any γ_R, the maximum speed to windward is determined by the condition that

$$\partial v_{MG}/\partial P = 0 \tag{17}$$

This condition leads to a relationship between γ_R and P given by

$$\cos\gamma_{R_0} = 2P_0/(P_0^2 + 1) \tag{18}$$

and the optimum speed to windward is given by

$$v_{MG_0} = 1/(P_0^2 - 1) \tag{19}$$

These relationships also are shown on Fig. 8.

Equation (16), taken with the condition expressed by (17), gives for the operating condition of the sail and hull

$$\partial\gamma_R/\partial C_{L_S} = 0 \qquad \partial\gamma_R/\partial C_{L_H} = 0$$

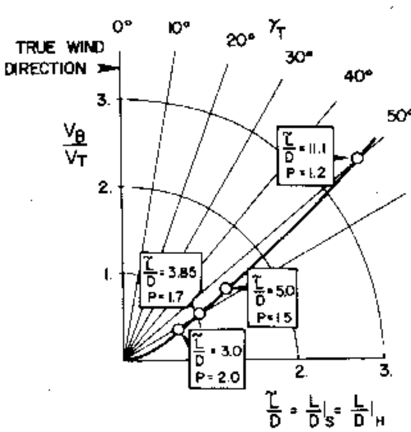

Fig. 9 Maximum speed to windward as a function of lift/drag ratio.

These conditions imply that each component is adjusted to operate at its maximum L/D. Furthermore, for given maximum lift/drag ratios of the sail and hull, the maximum speed to windward is obtained for a particular value of the performance parameter P given by Eq. (18). Figure 9 shows these results on a polar graph and also indicates the value of P corresponding to various lift/drag ratios. Note the impressive windward performance predicted by this result. If, in the design of the craft, lift/drag ratios of 5 can be achieved for the sail and hull, then $P_0=1.5$, and the craft will be sailing at 1.5 times the speed of the wind and making a speed to windward of 85% of the true wind, sailing at 55° from the true wind direction.

This result yields much better performance than is achieved by sailing craft. The lift/drag ratios assumed in this example are typical[1] so the reason why this impressive performance cannot be achieved must be related to physically possible values of the parameter P.

For a variety of small boats ranging from a Tempest to a Lightning, the square root of the density ratio times the centerboard to sail area ratio varies from about 4 to 6. Taking 5 as an average value for this ratio, to achieve a value of P equal to 1.5 would require a ratio of force coefficients of the order 0.1 at (L/D)max. This appears to be very difficult to achieve, particularly with respect to the low lift coefficient required of the hull at maximum lift/drag. In the example given on p. 322 of Ref. 1, $\gamma_{R_M}=21.5°$ for a V_B of 3.5 kts and the corresponding P_M is 3.18. P_0 for this example is given by Eq. (18) as 1.45.

Since it appears, from a practical standpoint, difficult to achieve the value of P required by Eq. (18), the more general condition given by Eq. (16) must be examined. In general, P_M corresponding to realistic values of γ_{R_M} will be greater than P_0, and in this case $\partial v_{MG}/\partial P$ will be negative, and therefore, the left-hand side of Eq. (16) will be positive. Since $\partial P/\partial C_{L_H}>0$ and $\partial P/\partial C_{L_s}<0$, the best speed to windward is obtained when $\partial\gamma_R/\partial C_{L_s}>0$ and $\partial\gamma_R/\partial C_{L_H}<0$, indicating that the sail should be trimmed for a lift coefficient above that for maximum L/D and the hull should be trimmed for a lift coefficient below that for maximum L/D for the best speed to windward.

Also, it can be seen that a necessary condition for the optimum (maximum speed to windward) which relates the operating condition of the sail to the operating condition of the hull is given by

$$\frac{\partial P}{\partial C_{L_H}}\frac{\partial \gamma_R}{\partial C_{L_s}}=\frac{\partial P}{\partial C_{L_s}}\frac{\partial \gamma_R}{\partial C_{L_H}} \tag{20}$$

Equation (20) is a relationship between P and γ_R in terms of the characteristics of the sailing craft which must be compared with the wind triangle relationships to determine the condition for maximum speed to windward. However, the operating condition of the craft for maximum speed to windward must lie on the line determined by Eq. (20). A typical curve given by Eq. (20) is shown in Fig. 10 with $P_M>P_0$. The curve given by Eq. (20) is a boundary on possible windward sailing conditions and the trim condition of the craft must lie inside or on this boundary. The maximum speed made good occurs when this curve is tangent to a line of constant speed made good.

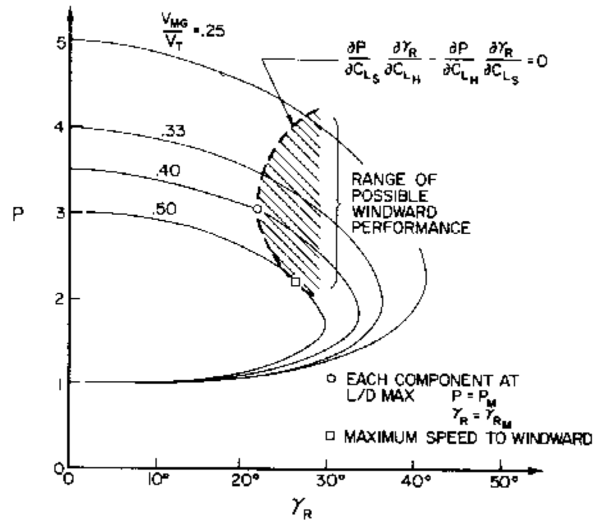

Fig. 10 Windward performance boundaries.

In terms of lift and drag coefficients, the necessary condition given by Eq. (20) is

$$\frac{(dC_D/dC_L)_H-(C_D/C_L)_H}{I+[(dC_D/dC_L)_H(C_D/C_L)_H}=\frac{(C_D/C_L)_s-(dC_D/dC_L)_s}{I+[(dC_D/dC_L)_s(C_D/C_L)_s]} \tag{21}$$

This condition may be stated for an efficient craft, that is, when the denominators of Eq. (21) are approximately equal to one as

$$\frac{dC_D}{dC_L}\Big|_H+\frac{dC_D}{dC_L}\Big|_s\cong\frac{C_D}{C_L}\Big|_s+\frac{C_D}{C_L}\Big|_H \tag{22}$$

To obtain more specific results assume that the drag coefficients vary as the square of the lift coefficients, so that $C_D=C_{D0}+C_{D_i}$ and $C_{D_i}=KC_L^2$.

Introducing this relationship into Eq. (22) yields

$$\frac{C_{D_i}}{C_L}\Big|_H+\frac{C_{D_i}}{C_L}\Big|_s\cong\frac{C_{D_0}}{C_L}\Big|_s+\frac{C_{D_0}}{C_L}\Big|_H \tag{23}$$

Thus, a necessary condition for maximum speed to windward, approximately given by Eq. (23), states that the sum of the induced drag/lift ratios of the hull and sail is equal to the sum of the profile drag/lift ratios.

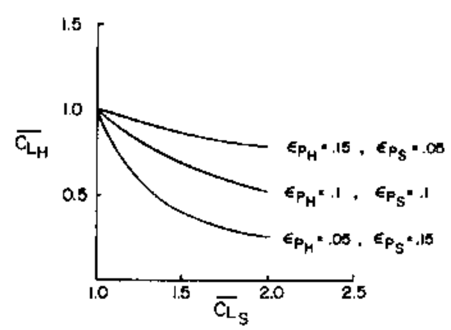

Fig. 11 Relationship between sail lift coefficient and hull lift coefficient necessary for maximum windward performance.

To summarize, Eq. (20) determines a curve which bounds the windward performance, and the maximum speed to windward lies somehwere on this line, precisely at the point where this line is tangent to a constant v_{MG} line as stated by Eq. (16).

Performance Tradeoffs

Now some of the implications of these results on windward performance are examined. Expressing Eq. (20) in terms of $\bar{C}_L$, the ratio of the lift coefficient to the lift coefficient at (L/D)max, and ϵ_p, the profile drag/lift ratio at (L/D)max

$$\frac{\epsilon_{p_s}(\bar{C}_{L_s}^2-1)}{\bar{C}_{L_s}[1+2\epsilon_{p_s}^2(\bar{C}_{L_s}^2+1)]}=\frac{\epsilon_{pH}(1-\bar{C}_{LH}^2)}{\bar{C}_{LH}[1+2\epsilon_{pH}^2(\bar{C}_{LH}^2+1)]} \tag{24}$$

The drag/lift ratios are

$$\epsilon_H=\epsilon_{pH}\left[\frac{1}{\bar{C}_{L_H}}+\bar{C}_{L_H}\right] \tag{25a}$$

$$\epsilon_s=\epsilon_{p_s}\left[\frac{1}{\bar{C}_{L_s}}+\bar{C}_{L_s}\right] \tag{25b}$$

where

$$\tan\gamma_R=(\epsilon_H+\epsilon_s)/(1-\epsilon_H\epsilon_s) \tag{26}$$

and

$$P=\left(\frac{\rho_w S_H}{\rho_A S_s}\right)^{1/2}\left(\frac{C_{LHM}}{C_{LsM}}\right)^{1/2}\left(\frac{\bar{C}_{LH}}{\bar{C}_{L_s}}\right)^{1/2}\frac{(1+\epsilon_H^2)^{1/4}}{(1+\epsilon_s^2)^{1/4}} \tag{27}$$

If the profile drag/lift ratios at (L/D) max are given, then Eq. (24) determines the relationship between the lift coefficients of each component ratioed to the values of C_{LM} of each component. Figure 11 shows the variations of $\bar{C}_{LH}$ with $\bar{C}_{L_s}$ given by Eq. (24) for various values of ϵ_p. Only $\bar{C}_{L_s}$ larger than 1 is shown since this corresponds to the lower branch of the boundary curve of Fig. 10. Then the relationship between γ_R and P can be determined from Eqs. (26) and (27). This relationship is shown in Fig. 12 where

$$\frac{P}{P_M}=\left(\frac{\bar{C}_{LH}}{\bar{C}_{L_s}}\right)^{1/2}\frac{(1+\epsilon_H^2)^{1/4}}{(1+\epsilon_s^2)^{1/4}}$$

The actual performance of the craft is determined by the centerboard area S_H, the sail area S_s, and the ratio of the lift coefficients corresponding to the maximum lift/drag ratios, as can be seen from Eq. (27). If these areas are given, the trends of Figs. 10 and 12, taken with the fact that P_M is greater than the optimum value, illustrate that increased windward performance is achieved by operating the sail at a lift coefficient above that for (L/D) max, and the hull at the lift coefficient below that for (L/D) max, as noted before. On the basis that the value of P at maximum lift/drag of each component

$$P_M=\left(\frac{\rho_w S_H}{\rho_A S_s}\right)^{1/2}\left(\frac{C_{LHM}}{C_{L_{sM}}}\right)^{1/2} \tag{28}$$

will tend to be greater than the optimum value P_0, increased windward performance can be obtained by increasing the sail area. Note that this would not be the case if $P_M=P_0$. Also, this expression indicates that reducing the centerboard area will improve performance. Care should be taken with this interpretation however, since reducing the centerboard area increases the hull minimum drag coefficient owing to the fact that the centerboard area was used for non-dimensionalization. The profile drag coefficient of the hull is

$$C_{D_{0H}}=[2+(S_w/S_H)]C_{D_w}$$

and therefore, reducing the centerboard area increases the hull drag coefficient and it is less clear that this results in a performance increase.

Since P_M determines the location of the curve shown in Fig. 12 on the speed made good to windward graph of Fig. 10, some indication of the performance tradeoffs can be examined further in terms of the movement of P_M with changes in the parameters of the craft. Based on the aerodynamic and hydrodynamic assumptions made

$$C_{L_M}=(C_{D_0}/K)^{1/2} \quad \epsilon_p=(C_{D_0}K)^{1/2} \tag{29}$$

where $K\propto(1/AR)$. Equation (28) taken with the condition that $P_M>P_0$ implies that C_{LHM} should be as small as possible and that $C_{L_{sM}}$ should be as large as possible.

If the minimum drag/lift ratio of each component is considered to be given (ϵ_p = constant), Eq. (29) shows that good windward performance is achieved if the minimum hull drag/lift ratio is a result of a low drag coefficient and the sail minimum drag/lift ratio is a result of high aspect ratio, as this results in a high C_{L_M} for the sail and a low C_{L_M} for the hull. In the opposite case, that is, with the same minimum drag/lift ratios achieved by a high aspect ratio centerboard and low aspect ratio sail with corresponding changes in the drag coefficients, the windward performance will be poor. As shown in Fig. 13 increasing the centerboard aspect ratio or reducing the sail profile drag coefficient moves P_M upward and to the left tending to reduce the speed made good to

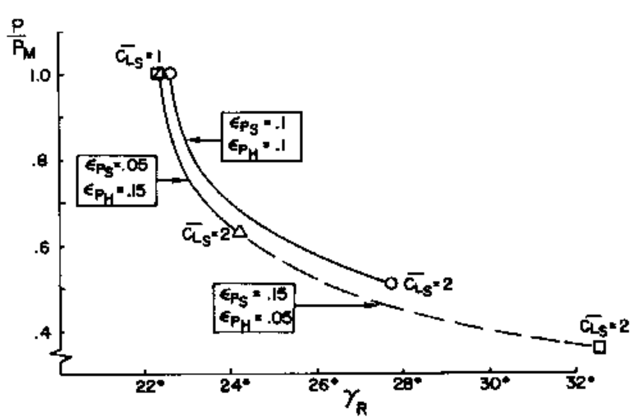

Fig. 12 Relationship between P/P_M and relative wind angle necessary for maximum windward performance.

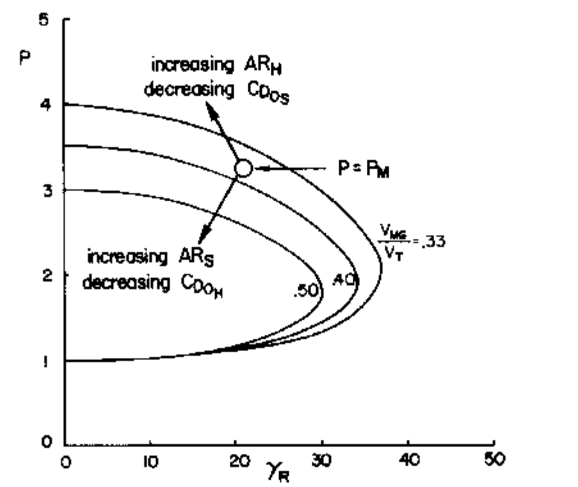

Fig. 13 Variation of P_M with aspect ratio and profile drag coefficient.

windward. In contrast, reducing the hull drag coefficient or increasing the sail aspect ratio clearly moves P_M in a favorable direction.

In general, movement of the P, γ_R curve indicates a gross trend; however, care should be taken with the interpretation that increasing the profile drag coefficient of the sail will improve windward performance. The actual change in performance depends upon how far along the P, γ_R curve it is possible to move by retrimming the craft. To obtain increased windward performance by increasing the sail drag coefficient, there is an implication that a very high sail lift coefficient is required to achieve this improvement as may be seen from Fig. 12, and Eq. (29). Also, since a high sail lift coefficient is required, the assumption regarding the variation of sail drag coefficient with the square of the lift coefficient becomes questionable.

This approach appears to show considerable potential for understanding the complex interactions involved in sailing to windward, and presents a convenient method for studying the influence of design tradeoffs on windward performance.

Conclusions

1) The wind triangle relationships for a sailing craft with zero heel angle can be presented so that the geometry is related directly and simply to the aerodynamic and hydrodynamic characteristics of the craft.

2) Given the maximum lift/drag ratios of the sail and hull, there is a specific value of the performance parameter P which will maximize the speed made good to windward.

3) For typical lift/drag ratios of the sail and hull, it appears difficult to achieve the corresponding value of the performance parameter P which maximizes the speed made good to windward. As a result, the best windward performance is achieved by trimming a sailing craft so that the sail operates at an angle of attack above that for maximum lift/drag and the hull operates at a leeway angle below that for maximum lift/drag.

Reference

[1]Marchaj, C. A., *Sailing Theory and Practice*, Dodd, Mead and Company, New York, 1964.